Environ Econ Policy Stud (2018) 20:1–15
https://doi.org/10.1007/s10018-016-0179-9

RESEARCH ARTICLE

Weighted location differential tax in environmental problems

George E. Halkos[1] **· Dimitra C. Kitsou**[1]

Received: 1 March 2016 / Accepted: 7 October 2016 / Published online: 21 October 2016

Abstract Based on Pigou's view, environmental taxation increases the cost of polluting activities reflecting the true social cost imposed on societies by the environmental damage caused by these activities. Imposing an emissions tax is a standard way to internalize the external pollution damages into producers' decision-making. An efficient outcome is attained, when marginal external cost of emissions is identical for all producers and equal to marginal abatement cost of each producer. When producers are heterogeneous, however, a uniform emission charge usually fails to satisfy these requirements. In this case, ideally, taxes should be differentiated across pollution sources to consider variations in the marginal damage caused by their respective activities. In this paper, the total pollution cost is related with contaminated locations and a weighted-location-differentiated tax together with a corresponding index that adjusts taxation to the damages caused, is proposed. The weights follow a gamma-order normal distribution, which is described by shape, location and scale parameters, allowing for some flexibility in the measure of spatially differentiated environmental impacts.

Keywords Weighted-location-adjusted differential tax · Pollution-related social cost · Expected value · Technology · Probability density function

JEL Classification C02 · C60 · Q50 · Q53 · Q58

✉ George E. Halkos
halkos@uth.gr

[1] Laboratory of Operations Research, Department of Economics, University of Thessaly, Volos, Greece

1 Introduction

In the past, command and control regulations (like limiting the use of specific fuels or demanding certain pollution sources to use specific methods) dominated environmental policies with market-based instruments (like taxes and tradable permits) to govern over the last decades. Environmental taxation relies on Pigou's concept of increasing polluters' private costs to a level that includes the associated true social costs imposed on society by their activities and the resulting related environmental damages.

Thus, taxing emissions is a standard way to get producers to internalize the external pollution damages into their decision-making. An emission charge yields the efficient outcome if, and only if, it is such that the marginal external cost of emissions is identical for all producers and the marginal external cost of emission is equal to the marginal control cost of each producer. When producers are heterogeneous, however, a uniform emission charge usually fails to satisfy the above requirements. In this case, ideally, taxes should be differentiated across firms to consider variations in the marginal damage caused by their respective activities.

The problem is how to evaluate the deposits of pollution in an area far away from the source of pollution. In the geosciences, this may be done by interpolation with the pollution source i having a known (monitored) value but a deposition point j having an unknown value and being approximated from the values of pollution at source point i. In assessing the effect of pollution, measurements from (reference or sampled) remote stations are usually weighted by the inverse of distance raised to some nonnegative power (IDW). This stems from Shepard's method of spatial interpolation (Shepard 1968). Alternatively, we may have the kriging or optimal interpolation where the weights depend on the spatial correlation of the provided data at monitored stations (Krige 1951).

In the case of IDW, weights depend on the distance between monitoring stations and are exogenous to the data with the exponent chosen arbitrarily. In IDW, and by weighting each point by the inverse of the distance, the monitoring stations close to the source of pollution, as by being near to the sampled point they take an immense importance in the weighting and the resulting burden while more distant points are allocated less weighting burden. De Mesnard (2013) presents a number of critical remarks on pollution models and IDW, reviewing extensively the existing relative literature on interpolation variants applied in various research applications. Mesnard examined the subjective character of the distance exponent and the associated problem of monitoring stations close to the point of reference. In these lines, it is verified which distance exponent should be chosen depending on the form of pollution encountered, like radiant pollution, air pollution and polluted rivers. But a number of problems may be mentioned. The subjective character of the exponent of distance implies that weights are not established according to the type of pollution and for some pollutants, like global pollutants, distance does not matter.

Similarly, with respect to data the weights are endogenous in the case of optimal interpolation depending on the spatial correlation of the data recorded at monitoring stations. This makes kriging complex and computational demanding (Kerry and

Hawick 1998; Lloyd 2006; Pesquer et al. 2011) with the possibility of negative weights too (Deutsch 1996).

In our paper, instead of choosing an arbitrary exponent we propose the use of appropriate ratios. At the same time we give importance to the abatement effort (if any) in the source or sampled stations. A weighted-location-adjusted differentiated taxation is introduced, based on the principle that when pollution is above "an optimal and acceptable level" higher taxation has to be imposed, while if it is below there is a chance of lower taxation. In this way a new index to adjust taxation to the damage caused is proposed.

The structure of the paper is as follows: Section 2 reviews the relative existing literature, while Sect. 3 explains and defines the proposed weighted-location-adjusted differential taxation. Section 4 discusses the use of the appropriate distribution and the evaluation of the expected value and the variance of the total pollution social cost. The last section concludes the paper and refers to the associated policy implications of the proposed tax differentiation.

2 A brief review of existing relative literature

Economic theory indicates that the optimal tax rate is determined where marginal abatement cost (MAC) equals to marginal damage cost (MD) imposed by emissions. The problem of relating taxation and pollution has been considered by many researchers (among others, Chen and Liu 2005; Akao 2008; Numata 2011; Akao and Managi 2013; Kampas and Horan 2016). The target is an equitable sharing of charges on polluters. Such a model could be used, for example, in harbors with heavy traffic, where the entrance or exit of ships pollutes the environment corresponding to the quality of the vessel. Therefore, the technology used, will be associated with the taxation system. A new tax, which depends on innovation and at the same time, is above the expectations of a Pigouvian analysis was proposed by Requate (2005). The proposal of Requate under stochastic innovation has the same importance as the analysis of the other environmental measures.

As the distance and the location of GHG emissions' sources are not related to the location of the environmental damages and degradation, they are considered as uniformly mixing pollutants[1] with their concentration levels to be invariant from place to place. In the case of uniformly mixing pollutants, the pollution levels depend on their total emissions levels. Similarly, in the case of non-uniformly mixing pollutants, locations of their emission sources are significant in determining the spatial distribution of ambient levels of pollution (Perman et al. 2003). In the case of non-uniformly mixed damage (i.e., health and environmental damages from pollution depending on the location of the source), efficiency demands that the marginal costs of emissions control should be different across pollution sources and should be determined by the damage caused (Montgomery 1972; Tietenberg 2006).

[1] Uniformly mixing pollutants take place when physical processes function to disperse them to the point where their spatial distribution is uniform (Perman et al. 2003, p. 178).

Moving from a uniform (undifferentiated) to a differentiated policy implementation will obviously increase welfare provided that parameters are set optimally and, as mentioned already, damages differ across pollution sources. This may be accomplished by considering the associated marginal damages imposed across sources. Coping with this issue we extend the existing results and propositions and introduce the differentiated weighted-location-adjusted tax.

Uncertainties in abatement and damage cost functions affect policy design in various ways. Pollution control and damage cost functions are non-linear and their exact shapes are usually unknown (Halkos and Kitsos 2005; Halkos and Kitsou 2014). Firms do not have always an incentive to reveal their true abatement costs.[2] At the same time, environmental effects are associated with significant irreversibility interacting often in a very complex way with uncertainty. This complexity becomes even worse when considering the very long-run character of many environmental problems. When marginal abatement costs are known and constant, the policy maker of the environmental issues (for instance the local authorities) can minimize social cost by introducing a pollution tax that equalizes marginal abatement and damage costs.

In the absence of information about costs, the level of emissions taxes needed to achieve a target level of pollution abatement is unknown. This problem can be overcome by using an iterative procedure in which tax is adjusted. The tax that its tax system results in the social optimal pollution level is the differential tax. With differential taxation, the marginal emission tax paid by firm i is always equal to marginal damage costs and thereby minimizing social costs. The reason why this tax system results in the social optimal pollution level is that the firms—faced with a tax level that depends on emissions of firms—have an incentive to share information with respect to their abatement cost.

The analysis becomes more complicated when the abatement costs are stochastic, i.e., developed around it a probabilistic randomness. In this case or when we have changes in the marginal abatement costs, specific environmental policies are required, because the results from the changes of the Pigouvian taxation may be considered obsolete. Marginal abatement costs may change over time, by changing the innovative standards in the industry and by adopting the rapidly evolving new technologies. It is worth mentioning that adoption of new technologies decreases or aims to reduce emissions.

But in most cases, existing and planned emissions regulations are imposed as spatially uniform, "undifferentiated" policies with all regulated emissions being penalized at a uniform (same) tax rate or permit price (Fowlie and Muller 2013).

[2] Estimation of damage cost functions is much more complicated compared to abatement costs, as the influences of pollution cannot be identified with accuracy and there are many cases where it takes a long time to realize the effects of the damage imposed. To extract damage estimates in the case of acidification and the related transboundary pollution nature, a model taking account of the distribution of the externality among various countries (victims) is needed. As it is difficult to have a direct estimate the damage function its parameters may instead be inferred assuming countries equate national MD with national MAC and where restrictions on the derivatives of the damage cost function are significant. In this way the damage function may be "calibrated" assuming that national authorities act as Nash partners in a non-cooperative game with the rest of the world, taking as given deposits originating in the rest of the world (Hutton and Halkos 1995; Halkos 1996).

Theoretically, market-based policies may tackle non-uniformly mixed pollutants (like NO_X, SO_2) with the optimal tax to be calculated by the marginal damage imposed. Taxes are different by pollution source for different levels of damage imposed. Differentiation will be profitable depending on the variation in damages caused across sources as well as the slopes of MACs (Mendelsohm 1986; Halkos 1993, 1994; Fowlie and Muller 2013). Long and Soubeyran (2005) revealed that the tax rates per pollutants unit are not identical for all producers in an competitive market and label their findings as "property of selective penalization".

In general, almost all tax systems involve differentiated tax rates among the various sectors (industry, commerce, households etc.). In the case of uniform taxation the same marginal abatement costs are assumed with the economy in total to use the cheapest pollutant control methods in each sector. The reduction in the tax rate in a sector, and in order to attain the imposed environmental target, may increase the taxes imposed in other sectors. This implies that any deviation from uniform taxation may impose excess costs. Thus differentiated taxation among different sectors of an economy is optimal due to, among others, initial tax distortions, distributional concerns, trade terms and leakage motives (Böhringer and Rutherford 2002).

As it is known, in a first-best policy, taxes should be differentiated between pollution sources according to the size of their resulting damage costs. A second-best policy relies on the imposition of a high uniform tax rate. Halkos (1993) showed that moving from the first-best optimum to a uniform tax rate does make a difference. Specifically and in the case of the acid rain problem in Europe it was shown that the costs of moving from the first-best to the imposition of a high uniform tax rate may not differ so much across countries but may be quite different within countries.

Pre-existing tax distortions influence the efficiency effects of newly imposed environmental taxes. Among others, Bovenberg and van der Ploeg (1994), Bovenberg and Goulder (1996) and Goulder et al. (1997) propose that tax interaction leads to higher efficiency costs (net of environmental benefits) of environmental taxation compared to a first-best case leading to optimal second-best environmental tax rates lower than the Pigouvian rate. Agrawal (2012) addresses whether it is optimal in the case of multiple regions to impose differentiated taxes across the regions and concludes that maximization of social welfare requires geographical tax differentiation.

According to Mazzanti and Zoboli (2013) in addition to economic instruments the formation and monitoring of environmental policies is important in environmental planning. They highlight that to achieve sustainability the land use has to be planned with appropriate economic instruments considering the two as complements. Akao (2014) discusses the preference constrain for a sustainable development path relying on the need of clean economic growth, the high assimilative capacity of the environment to support growth and the population with high ($\geq$1) marginal utilities of consumption. Additionally, Cremer et al. (2004) examined the political support as important determinant in environmental taxation planning and their effectiveness. At the same time, revenues raised by the imposition of environmental taxes may be used to reduce the distortions of the existing taxes

(Terkla 1984; Oates 1995) offsetting in this way part of potentially negative tax interaction effects (Goulder 1995).

That is why we adopt the generalized γ-order Normal distribution for the analysis bellow. This distribution is based on an extra, shape parameter γ, which under different values of γ coincides with a number of well known distributions. Among them, and as it will be shown in the next section, with $\gamma = 1$ is the Uniform distribution, with $\gamma = 2$ is the well known Normal distribution and with $\gamma =$ infinity, practically very large (or very small) coincides with the Laplace distribution.

3 The weighted-location-adjusted differential tax

Given the abatement technology level (X), the Total Pollution Cost (TPC) is defined by the sum of the total abatement (TAC) plus the total (external) damage (TD) costs. That is the random variable TPC includes the *social* costs associated with pollution. In this paper we evaluate the expected value of TPC and introduce the estimation of its variance. Specifically, choosing as TPC the general form TPC $= (\kappa X + \lambda)^2$ (with κ, λ constants and X the current level of abatement technology) coming from the γ-order generalized normal distribution[3] we provide a generalization of the E(TPC) both in the form of TPC and the probability density function. The current level of abatement technology represented by X is related with the emissions distribution described by a *shape* parameter, a *location* parameter (the center of the pollution) and a *scale* parameter (the variance of pollution concentration around the center of pollution).

In this way we propose a "weighted location differential tax" to existing tax systems and a corresponding ratio to provide us with an index adjusting taxation to the damage imposed. Theoretically this tax will be non-linear (since high pollutants should face appropriate taxes i.e., exponential greater and not linear) and non–time consistent (as pollution is not time constant depending for instance on weather conditions, amount of production, etc.). This new indicator for environmental policy is based on a generalization of the differential taxation (Halkos 1993, 1994; Kim and Chang 1993; McKitrick 1999) and provides another look of differentiation in taxation, based on the location and the assumed distribution the new introduced technologies follow.

Our argument is that around the pollution center (*source of pollution*) pollution is distributed according to a (possible) statistical model, related with the actual situation. In such a case it may be uniformly distributed i.e., in a distance, left or right from the pollution center the pollution to remain constant. That might be mathematically a helpful assumption but it is difficult to be true. Another approach is to consider a normally distributed pollution dispersion, with the mean being at the pollution center, so plus or minus it one standard deviation concentrates approximately the 0.68 of the pollution. In the cases of a 0.99 level of pollution concentration we may consider a $\pm 3\sigma$ confidence interval (or $L = 6\sigma$) as essential.

[3] For more information on the γ-order generalized normal distribution see Appendix 1.

This is near to be true, as the tails contain a very small probability level to allow a pollution influence.

Similarly, the Laplace distribution offers a solution to provide a "strong" pollution center and fat tails. All these three distributions are special cases of the γ-order Generalized Normal distribution.[4] In this particular distribution, the third involved parameter, the shape one, called γ, taking all real values, but not within [0, 1], offers a number of different distributions with fat tails mainly. With the value of $\gamma = 1$, it is reduced to Uniform; with the value $\gamma = 2$ is reduced to Normal; with the value of γ "infinity" practically very large is Laplace. In Sect. 4, we obtain the appropriate evaluations for the total pollution (social) cost (TPC).

Now, having the expected value and the variance of the total pollution cost, E(TPC) and Var(TPC), approximate 95 % confidence intervals (CI) can be obtained—which are precise only in the Normal case of the form

$$\mathrm{CI(TPC)} = \mathrm{E(TPC)} - 2\,(\mathrm{Var(TPC)})^{0.5},\ \mathrm{E(TPC)} + 2\left(\mathrm{Var(TPC)}^{0.5}\right) \tag{1}$$

The length of this 0.95 confidence interval is $L = 4[\mathrm{Var(TPC)}]^{0.5}$. Similarly and in the case of a 99 % CI, as mentioned, we work with the "distance" D of the end points of $\pm 3\sigma$ (or 6σ) CI with $D = 6[\mathrm{Var(TPC)}]^{0.5}$ a kind of *Quality Control criterion* of the pollution. That is how far from the center of pollution the area is contaminated with a 99 % probability. Obviously higher levels of TPC may involve higher abatement levels with variable X having to adjust to a continuous search for more cost-effective existing or new control methods. The application of these methods is expected to contribute to more abatement and less external costs imposed on the society.

When pollution is at the optimal level the optimal length as above is L^* or D^*. Usually we refer to the expected level of abatement required by international conventions and other Protocols but what is real is the actual level of current abatement. Therefore the ratio

$$\Delta(\mathrm{Tax}) = \frac{L}{L^*} \text{ or } \Delta(\mathrm{Tax}) = \frac{D}{D^*} \tag{2}$$

is essential and can be a fair index to provide a weighted-location-differentiated taxation, as the case $\Delta(\mathrm{Tax}) > 1$ is expected to be faced in existing tax systems. The tax burden will be determined using expression (2) which depends on the optimal level of pollution L^* or D^* based on the choice of the appropriate (new) abatement method X and the corresponding TPC. More simply, with L^* and D^* we denote the optimal cases where the variance of TPC that is the variability of pollution is as expected and as a consequence the confidence limits are also expected. L and D may be the real length of the confidence intervals for 95 and 99 % respectively. That is the corresponding ratio as in (2) provides researchers with an index adjusting taxation to contamination caused. If the evaluated in each case L and D are less that the optimal then the tax burden will be less. In such a way a source of pollution (industry, firm, etc.) has an incentive to look for more efficient control methods.

[4] See Appendix 1.

This idea can be also adopted when the pollution center (that is the pollution source point) might be moving, as an aeroplane or a boat. In such a case around the pollution center a "sphere" of pollution is created of the form

$$(x-a)^2+(y-b)^2+(z-c)^2=R^2 \tag{3}$$

With R being the radius of the sphere and $K(a, b, c)$ the pollution center. If $R > R^*$, with R^* being the optimal pollution level radius, a weighted-location-differentiated taxation is needed, in the sense that a radius of pollution R is accepted, based on the adopted technologies, but beyond that, there is a problem. In Sect. 4 we proceed with the evaluation of the expected value and the variance of the total pollution cost.

4 Adopting the appropriate distribution

The easiest way, as far as the mathematical calculations are concerned, despite its unrealistic character, is to assume that the stochastic variable X—as a result of the R&D procedure,[5] is uniformly distributed in the interval $\left[\frac{1}{2}-\delta, \frac{1}{2}+\delta\right]$, say, recalling the definition of the Uniform distribution. This means that in this research we suppose eventually the variable TPC is derived from the Uniform distribution, i.e. $u(\frac{1}{2}-\delta, \frac{1}{2}+\delta)$ implying a uniform density function for X of the form

$$f(X)=\frac{1}{2\delta} \text{ for } X \in \left[\frac{1}{2}-\delta, \frac{1}{2}+\delta\right] \tag{4}$$

From the definition of the expected value the pollution related t- social cost for the linear tax equals to

$$\mathrm{E}[\mathrm{TPC}_{t_l}] = \int_{\frac{1}{2}-\delta}^{\frac{1}{2}+\delta} \mathrm{TPC}\, f(x)dx$$

Any general form of $\mathrm{TPC} = (\kappa X + \lambda)^2$ is presenting the appropriate area for TPC.

An extension of the calculation of expected value is needed as it can be either normal with the known tails or a "sharp" one around 'center' with 'heavy tails', a Laplace distribution among others. Therefore the γ-order generalized Normal distribution was adopted[6] as the extension of the Uniform distribution. The expected value of TPC can be evaluated and it can be seen that that the distribution is not only the Uniform but the $\mathrm{N}_\gamma(\mu, \sigma^2)$.[7] Figure 1 clarifies the generalization and represents

[5] Managi et al. (2016) discuss the uncertainties of investment costs and cash flows in the development of novel products' development.

[6] See Kitsos and Tavoularis (2009), Kitsos et al. (2012), Halkos and Kitsou (2014).

[7] See Appendix 1.

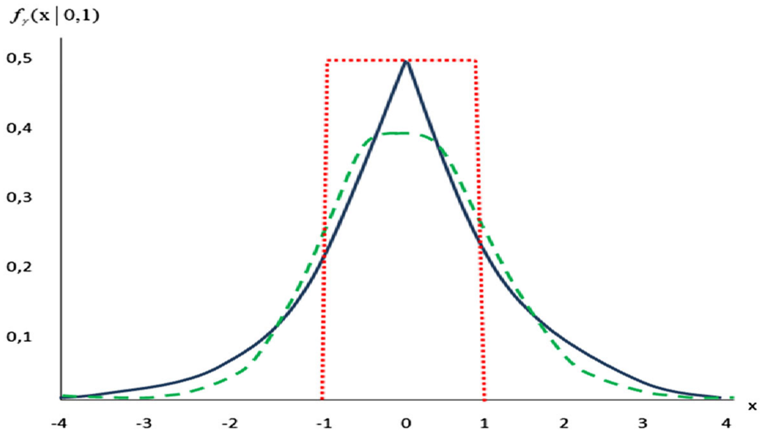

Fig. 1 Graphical presentation of the relationship between uniform, normal

the relation between Uniform, Normal and Laplace. This distribution regards a number of other distributions which are with 'fat tails'[8] and can be used in various economic analyses like for instance in stock markets. So, the following results are proposed for the form $(\kappa g + \lambda)^2$ and $f_\gamma(x; \mu, \Sigma)$ letting X represent a random variable describing innovation in searching for more efficient abatement methods.

As has been shown in Halkos and Kitsou (2014), if $\mathrm{X} \sim \mathrm{N}_\gamma(\mu, \sigma^2)$ it holds that:

$$\mathrm{E}\left[(\kappa \mathrm{X} + \lambda)^2\right] = (\frac{\gamma}{\gamma - 1})^{2^{\frac{\gamma-1}{\gamma}}} \frac{\Gamma(3\frac{\gamma-1}{\gamma})}{\Gamma(\frac{\gamma-1}{\gamma})}(\kappa\delta)^2 + \kappa\mu(\kappa\mu + 2\lambda) + \lambda^2 \tag{5}$$

$$Var((\kappa \mathrm{X} + \lambda^2)) = (\frac{\gamma}{\gamma - 1})^{4^{\frac{\gamma-1}{\gamma}}}(\kappa\delta)^4\left[\frac{\Gamma(5\frac{\gamma-1}{\gamma})}{\Gamma(\frac{\gamma-1}{\gamma})} - 4\frac{\Gamma^2(3\frac{\gamma-1}{\gamma})}{\Gamma^2(\frac{\gamma-1}{\gamma})}\right] - (\kappa\mu)^3(\kappa\mu + 4\lambda)$$
$$+ 2(\kappa\delta)^2\left[2\lambda^2 - (\kappa\mu)^2 - 2\kappa\lambda\mu\right](\frac{\gamma}{\gamma - 1})\frac{2\gamma - 1}{\gamma}\frac{\Gamma(3\frac{\gamma-1}{\gamma})}{\Gamma(\frac{\gamma-1}{\gamma})} \tag{6}$$

With different values of κ and λ a number of calculations for the corresponding TPC can be obtained. Next we present a number of examples.

As an example let us assume that $\mathrm{TPC} = (1/4 - 3/8X)^2$. Then it holds:

$$\mathrm{E}[\mathrm{TPC}_{\hat{t}_\ell;\gamma}] = \frac{1}{4} + 9(\frac{\gamma}{\gamma - 1})^{2^{\frac{\gamma-1}{\gamma}}}\frac{\Gamma(3\frac{\gamma-1}{\gamma})}{\Gamma(\frac{\gamma-1}{\gamma})}\delta \tag{7}$$

[8] This is why we are referring to "a family of distributions".

$$\mathrm{Var}\left(\mathrm{TPC}_{\hat{t}_\ell;\gamma}\right)=\frac{1}{4}\left(\frac{\gamma}{\gamma-1}\right)^{4\frac{\gamma-1}{\gamma}}(6\delta)^4\left[\frac{\Gamma\left(5\frac{\gamma-1}{\gamma}\right)}{\Gamma\left(\frac{\gamma-1}{\gamma}\right)}-4\frac{\Gamma^2\left(3\frac{\gamma-1}{\gamma}\right)}{\Gamma^2\left(\frac{\gamma-1}{\gamma}\right)}\right]+954\delta^2\left(\frac{\gamma}{\gamma-1}\right)^{2\frac{\gamma-1}{\gamma}}\frac{\Gamma\left(3\frac{\gamma-1}{\gamma}\right)}{\Gamma\left(\frac{\gamma-1}{\gamma}\right)}+13\frac{27}{4} \tag{8}$$

For this particular TPC, it holds that the expected value and variance of TPC can be evaluated for the Uniform, Normal and Laplace distributions as:

$$\mathrm{E}[\mathrm{TPC}_{\hat{t}_\ell;\gamma}]=\begin{cases}\frac{1}{4}+3\delta, & \text{Uniform}, \gamma=1,\\ \frac{1}{4}+9\delta, & \text{Normal}, \gamma=2,\\ \frac{1}{4}+18\delta, & \text{Laplace}, \gamma=\pm\infty,\end{cases} \tag{9}$$

$$\mathrm{Var}\left(\mathrm{TPC}_{\hat{t}_\ell;\gamma}\right)=\begin{cases}13\frac{27}{4}+318\delta-\frac{11}{45}(36\delta)^2, & \gamma=1, \text{Uniform}\\ 13\frac{27}{4}+954\delta-(18\delta)^2, & \gamma=2, \text{Normal}\\ 13\frac{27}{4}+1908\delta+2(6\delta)^2, & \gamma=\pm\infty, \text{Laplace}\end{cases} \tag{10}$$

From (9) it obviously holds that the quantity $\mathrm{E}\left[\mathrm{TPC}_{t_l,\gamma}\right]$ in the case of Uniform distribution is less than the corresponding Normal distribution, which is less that the corresponding Laplace distribution. That is:

$$\mathrm{E}\left[\mathrm{TPC}_{t_l,1}\right]<\mathrm{E}\left[\mathrm{TPC}_{t_l,2}\right]<\mathrm{E}\left[\mathrm{TPC}_{t_l,\pm\infty}\right]$$

For (10) and for $0<\delta<49.074$ it holds that[9]:

$$\mathrm{Var}^U(\mathrm{TPC})<\mathrm{Var}^N(\mathrm{TPC})<\mathrm{Var}^L(\mathrm{TPC})$$

Figure 2 shows that with $\gamma=1$ (the case of uniform) the expected value is less than in the case of $\gamma=2$ (the case of normal) and flatter compared to the other two cases. Similarly the results for the comparison between $\gamma=2$ (Normal) and $\gamma=\pm\infty$ (the case of Laplace) show that Laplace is sharper among them. This implies that changing D the expected value of TPC in the case of the uniform distribution is more stable compared to the other cases. With Laplace being the most sensitive in changing parameter δ, a small change in δ causes a sharp change in the expected value.

5 Discussion and policy implications

Environmental taxes should be targeted at the pollutants and be related to the environmental damage caused. Without any government intervention countries (firms) will not consider any environmental damage caused as this may be either spread across different regions or countries (as in the case of transfrontier pollution) or may be accumulated (stock pollution). For instance GHG emissions from one location may play an important role in global climatic changes.

[9] Halkos and Kitsou (2014) evaluate the differences between the variances of uniform, normal and Laplace to see, which ones are positive or negative, so that to rearrange the order among them.

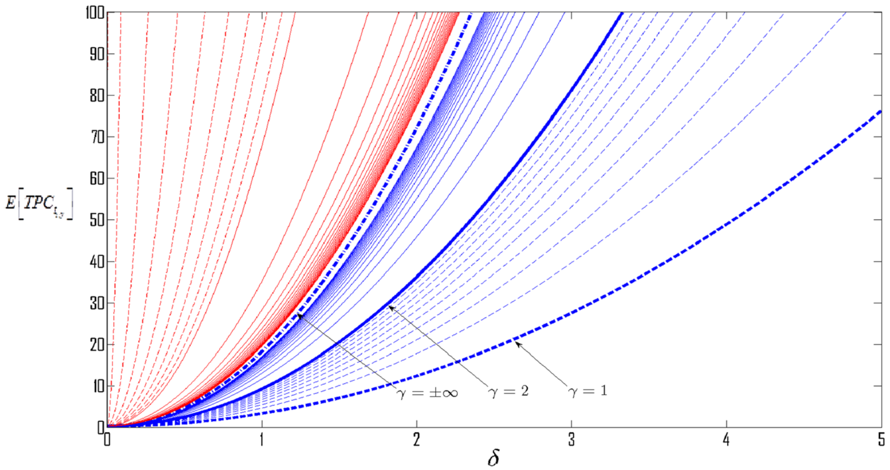

Fig. 2 Graphical presentations of $\mathrm{E}[\mathrm{TP}C_{t_\ell;\gamma}] = \mathrm{E}\ (1/4 - 3/8X)^2$ with $X_\gamma \sim \mathcal{N}_\gamma(\mu, \delta^2)$ as function of the scale parameter δ, for different values of the parameter γ (*blue* is for $\gamma \geq 1$ while *red* is for $\gamma < 0$)

In this paper we try to evaluate pollutants' deposits in areas far away from pollution sources. Instead of using spatial and optimal interpolations choosing arbitrary exponents we suggest the use of adequate ratios giving importance to the emission control effort in the pollution source. The way to cope with the problem is to tax directly the environmental damage costs due to the damages imposed. Thus, a weighted-location-adjusted differential tax is proposed together with a new suitable index to amend taxation to the damage created.

In particular, we have considered a general distribution that is followed by the random variable X of the (existing or new) abatement technologies adopted by a firm, and therefore the total pollution cost (TPC), covering three different lines of thought: a uniform approach of pollution around the center of pollution, adopting the new technologies; a normal that is most of the pollution around the center; and a 'sharp' portion of pollution around the center, i.e., the Laplace distribution.

The proposed weighted-location tax is differentiated according to the "level of distance" from the center of pollution i.e., how far from it has the area being contaminated due to this particular source of pollution. As a conclusion it is very clear that we are depending on the assumption of the distribution for the (stochastic) TPC variable.

In this paper as shown the application of the γ-ordered generalized Normal, which relies on the extra shape parameter γ with different values of γ coinciding with various well known distributions. Specifically, it provides to the researcher the option to choose among three distributions: The Uniform, Normal and Laplace. That is among no-tails, normal tails, and fat tails. The decision is also based on the value of δ we choose at the first step—'how far' from the 'origin of pollution' we go.

The question of what is the shape of the distribution to be followed is important. That is why the expected value of the total pollution cost, E(TPC), can be related to the appropriately calculated variance, Var(TPC), so that approximate 0.95

confidence interval of the form $\mathrm{E(TSC)} \pm 2\sqrt{\mathrm{Var(TPC)}}$ to be evaluated, while for a 0.99 approximate confidence interval, the factor 2 is replaced by 3. As the TPC includes the social cost related to pollution the greater expected value has to be associated with higher taxation under the weighted-location tax while the larger the variance the larger the area polluted and affected socially. Therefore, the taxation system should consider these issues.

Due to difficulties in having available reliable direct cost estimates this approach may be used with various sensitivity scenarios and existing sensitivity maps of ecosystems applied to various indirect effects of depositions (see for instance Kämäri et al. 1992). It is feasible for every country to estimate the area in a number of sensitivity classes with values determined by ecological criteria like geology, vegetation, soil type, rainfall amounts etc. For instance acidic depositions vary significantly with time and location.

If the relationship between source and receptor locations is not considered then the externality that is imposed will not be examined. The externality is considered by the appropriate consideration of the transfer coefficients as provided by the co-operative program for monitoring and evaluation of long-range transmission of air pollutants in Europe (European Monitoring and Evaluation Program, EMEP). Then mathematical models may be used by policy makers to define the optimal necessary emissions reductions for each pollution source (country) i and under the ecosystem sensitivity thresholds (see among others Halkos 1994).

Finally, the environmental regulator should address two different issues: the total amount of emissions and the spatially differentiated impacts of pollution. Although we propose to address both issues with a single instrument, a spatially differentiated emission charge, it may be more practical to use a combination of economic instruments. One solution could be the use of a cap and trade program to set a limit on aggregate emissions combined with spatially differentiated emission standards to account for spatial variations in damages. Our purpose at this stage was to indicate the appropriate burden allocation in terms of spatial heterogeneity.

Acknowledgments This research has been co-financed by the European Union (European Social Fund—ESF) and Greek national funds through the Operational Program "Education and Lifelong Learning" of the National Strategic Reference Framework (NSRF)-Research Funding Program: Heracleitus II. Investing in knowledge society through the European Social Fund. We thank also three anonymous reviewers for helpful comments on an earlier draft of this paper. Any remaining errors are the authors' responsibility.

Appendix 1

The γ-ordered normal distribution

The normal distribution $\mathrm{N}(\mu, \sigma^2)$, with mean μ and variance σ^2, is defined as:

$$f(x) = \frac{1}{(2\pi)^{\frac{1}{2}}\sigma} \exp\left\{-\frac{1}{2\sigma^2}(X-\mu)^2\right\} \tag{11}$$

The multivariate generalization for a multivariate random variable with p-conditions, μ mean and matrix covariance Σ is compared with (11) resulting to:

$$\varphi(\chi) = \frac{1}{(2\pi)^{\frac{p}{2}}|\Sigma|^{\frac{1}{2}}}\exp\left\{-\frac{1}{2}(X-\mu)^{\mathrm{T}}\Sigma^{-1}(X-\mu)\right\} \tag{12}$$

We denote this with $N_p(\mu, \Sigma)$, $|\Sigma| = \det(\Sigma)$.

A more general form of the multivariate distribution was investigated with an extra shape parameter, introducing through Logarithm Sobolev Inequalities (LSI) a new family of univariate γ-ordered Normal distribution the $\mathrm{N}_\gamma^\rho(\mu, \Sigma)$, which generalizes the Normal Distribution $\mathrm{N}^\rho(\mu, \Sigma)$, through an additional parameter $\gamma \in \mathbb{R} - [0, 1]$ (Kitsos and Tavoularis 2009; Kitsos and Toulias 2010; Kitsos et al. 2012). The new generalized Normal distribution commonly referred as *γ-ordered Normal distribution*.

When $f(x)$ is the probability density function of a random variable $\mathrm{X} \sim \mathrm{N}_\gamma^\rho(\mu, \Sigma)$ then, compared with (12) above, $f(x)$ is defined as:

$$f_\gamma(x;\mu\Sigma) = C_\gamma^p|\det\Sigma|^{-\frac{1}{2}}\exp\left\{-\frac{\gamma-1}{\gamma}[Q(x)]^{\frac{\gamma}{2(\gamma-1)}}\right\} \text{ with } x \in \mathbb{R}^\rho \tag{13}$$

where $Q(x) = (x-\mu)\Sigma^{-1}(x-\mu)^{\mathrm{T}}$ as in (12) with the normality factor

$$C_\gamma^p = \pi^{-\frac{p}{2}}\frac{\Gamma(\frac{p}{2}+1)}{\Gamma(p\frac{\gamma-1}{\gamma}+1)}\left(\frac{\gamma-1}{\gamma}\right)^{p\frac{\gamma-1}{\gamma}} \tag{14}$$

Where if we set $\gamma = 2$, i.e. $\mathrm{N}_2^\rho(\mu, \Sigma)$ it follows that:

$$C_2^p = \pi^{-\frac{p}{2}}\frac{\Gamma(\frac{p}{2}+1)}{\Gamma(\frac{p}{2}+1)}\left(\frac{1}{2}\right)^p = (2\pi)^{-\frac{p}{2}} = \frac{1}{2\pi^{\frac{p}{2}}} \tag{15}$$

It holds that the multivariate γ-ordered Normal distribution $\mathrm{N}_\gamma^\rho(\mu, \Sigma)$ for order values of $\gamma = 1,\ 2\ \pm\infty$ coincides with

$$\mathrm{N}_\gamma^\rho(\mu,\Sigma) = \begin{cases} D^\rho(\mu) & \gamma = 0 \quad p = 1,2 \quad \text{Dirac distribution} \\ U^\rho(\mu,\Sigma) & \gamma = 1 \quad \text{Uniform distibution} \\ \mathrm{N}^\rho(\mu,\Sigma) & \gamma = 2 \quad \text{Normal distibution} \\ L^\rho(\mu,\Sigma) & \gamma = \pm\infty \quad \text{Laplace distibution} \end{cases}$$

$$E[(\kappa X+\lambda)^2] = \kappa^2 E[X^2] + 2\kappa\lambda E[X] + \lambda^2 = \kappa^2\left(\mathrm{Var}(X) + E^2[X]\right) + 2\kappa\lambda E[X] + \lambda^2$$

References

Agrawal DR (2012) Games within borders: are geographically differentiated taxes optima? Int Tax Public Financ 19:574–597

Akao KI (2014) Preference constraint for sustainable development. Environ Econ Policy Stud 16(4):343–357

Akao KI (2008) Tax schemes in a class of differential games. Econ Theor 35(1):155–174

Akao KI, Managi S (2013) A tradable permit system in an intertemporal economy. Environ Resour Econ 55(3):309–336

Böhringer C, Rutherford TF (2002) In search of a rationale for differentiated environmental taxes. Centre for European Economic Research (ZEW), Mannheim Discussion Paper No. 02–30

Bovenberg AL, Goulder LH (1996) Optimal environmental taxation in the presence of other taxes: general equilibrium analyses. Am Econ Rev 86(4):985–1000

Bovenberg AL, van der Ploeg F (1994) Environmental policy, public finance and the labor market in a second-best world. J Public Econ 55:340–390

Chen HC, Liu SM (2005) Tradeable-permit pollution control systems with and without commitment to auditing. Environ Econ Policy Stud 7:15–37

Cremer H, Donder P, Gahvari F (2004) Political sustainability and the design of environmental taxes. Int Tax Public Financ 11:703–719

De Mesnard L (2013) Pollution models and inverse distance weighting: some critical remarks. Comput Geosci 52:459–469

Deutsch CV (1996) Correcting for negative weights in ordinary kriging. Comput Geosci 22(7):765–773

Fowlie M, Muller N (2013) Market-based emissions regulation when damages vary across sources: What are the gains from differentiation? NBER Working Paper 18801. http://nature.berkeley.edu/~fowlie/Fowlie_Muller_submit.pdf

Goulder LH (1995) Environmental taxation and the double dividend: a readers' guide. Int Tax Public Financ 2:157–183

Goulder L, Parry IWH, Burtraw D (1997) Revenue-raising vs. other approaches to environmental protection: the critical significance of pre-existing tax distortions. RAND J Econ 28(4):708–731

Halkos GE (1993) Sulphur abatement policy: implications of cost differentials. Energy Policy 21(10):1035–1043

Halkos GE (1994) Optimal abatement of sulphur emissions in Europe. Environ Resour Econ 4(2):127–150

Halkos GE (1996) Incomplete information in the acid rain game. Empirica 23(2):129–148

Halkos GE, Kitsos C (2005) Optimal pollution level: a theoretical identification. Appl Econ 37(13):1475–1483

Halkos GE, Kitsou DC (2014) Uncertainty in optimal pollution levels: modelling and evaluating the benefit area. Forthcom J Environ Plan Manag. doi:10.1080/09640568.2014.881333

Hutton JP, Halkos GE (1995) Optimal acid rain abatement policy for Europe: an analysis for the year 2000. Energy Econ 17(4):259–275

Kämäri J, Amann M, Brodin YW, Chadwick MJ, Henriksen A, Hettelingh JP, Kuylenstierna JCI, Posch M, Sverdrup H (1992) The use of critical loads for the assessment of future alternatives for acidification. Ambio 21:377–386

Kampas A, Horan R (2016) Second-best pollution taxes: revisited and revised. Environ Policy Studies 18(4):577–597

Kerry KE, Hawick KA (1998) Kriging interpolation on high performance computers. Technical Report DHPC-035, Department of Computer Science, University of Adelaide, Australia

Kim JC, Chang KB (1993) An optimal tax/subsidy for output and pollution control under asymmetric information in oligopoly markets. J Regul Econ 5:193–197

Kitsos PC, Tavoularis KN (2009) Logarithmic Sobolev inequalities for information measures. IEEE Trans Inf Theory 55(6):2554–2561

Kitsos PC, Toulias LT (2010) New information measures for the generalized normal distribution. Information 1:13–27

Kitsos PC, Toulias LT, Trandafir PC (2012) On the multivariate γ-ordered normal distribution. Far East J Theor Stat 38(1):49–73

Krige DG (1951) A statistical approach to some mine valuations and allied problems at the Witwatersand. Master S Thesis in Engineering University of the Witwatersrand, Johannesburg, North Africa

Lloyd CD (2006) Local models for spatial analysis. CRC Press, Belfast

Long NV, Soubeyran A (2005) Selective penalization of polluters: an inf-convolution approach. Econ Theory 25:421–454. doi:10.1007/s00199-003-0433-9

Managi S, Zhang Z, Horie S (2016) A real options approach to environmental R&D project evaluation. Environ Econ Policy Stud 18(3):359–394

Mazzanti M, Zoboli R (2013) Resource taxation and regional planning: revenue recycling for local sustainability in the aggregates sector. J Environ Plan Manage 56(6):893–916

McKitrick R (1999) A Cournot mechanism for pollution control under asymmetric information. Environ Resource Econ 14:353–363
Mendelsohn R (1986) Regulating heterogeneous emissions. J Environ Econ Manag 13(4):301–312
Montgomery W (1972) Markets in licenses and efficient pollution control programs. J Econ Theory 5(3):395–418
Numata D (2011) Optimal design of deposit–refund systems considering allocation of unredeemed deposits. Environ Econ Policy Stud 13:303–321
Oates WE (1995) Green taxes: can we protect the environmental and improve the tax system at the same time. South Econ J 61(4):915–922
Perman R, Ma Y, McGilvray J (2003) Natural resource and environmental economics, 3rd edn., Longman
Pesquet L, Corte A, Pons X (2011) Parallel ordinary kriging interpolation incorporating automatic variogram fitting. Comput Geosci 37:464–473
Requate T (2005) Timing and commitment of environmental policy, adoption of new technology, and repercussions on R&D. Environ Resour Econ 31(2):175–199
Shepard D (1968) A two-dimensional interpolation function for computer mapping of irregular lyspaced data. Laboratory for Computer Graphics. Harvard University
Terkla D (1984) The efficiency value of effluent tax revenues. J Environ Econ Manag 11:107–123
Tietenberg T (2006) Emissions trading-principles and practice. Resources for the Future Press